

# Design optimization of piezoelectric energy harvester subject to tip excitation<sup>†</sup>

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# Abstract

This research proposes a new design for a cantilever-type piezoelectric energy harvester in which a free tip is excited by any rotary motion of mechanical devices. A coupled field finite element model for the harvester is constructed using ANSYS and verification study is performed. Design optimization on the shape of the harvester is done to maximize output power. The design optimization result shows excellent performance when compared to a simple rectangular cantilever or the well-known tapered cantilever. The design results are prototyped and their improved performances are experimentally attested.

Keywords: Energy harvesting; Piezoelectricity; PVDF; Vibration; Rotary motion; Tip excitation; Shape optimization

# 1. Introduction

Energy harvesting (also known as energy scavenging) is the energy conversion process by which electrical energy is harvested from various ambient energy sources (e.g., solar power, thermal energy, wind energy, noise, and vibration). Mechanical strain from machine vibration is one of the most available ambient energy sources easily found in civil structures, machines, or human bodies. Among the energy conversion principles to convert mechanical strain into electrical energy, piezoelectricity is known to be one of the most effective and practical ways [1].

Various types of piezoelectric energy harvesters have been studied which scavenge the wasted energy from human bodies and vibrating machines. Kymissis et al. [2] and Shenck and Paradiso [3] proposed an energy harvester using the insole of a sports training shoe. The harvester is composed of PVDF and PZT layers, and it generates electrical energy from the shoe's strain energy. Leland et al. [4] manufactured a piezoelectric energy harvester using PZT-5A4E. They tested the prototype and generated about 29.3 $\mu$ W using human induced vibration of a staircase. Feenstra et al. [5] developed an energy harvesting backpack that can generate electrical energy from the strap strain. They replaced the backpack strap with a mechanically amplified piezoelectric actuator to generate about 0.4 mW. Elvin et al. [6] presented a self-powered strain sensor capable of transmitting the sensory data wirelessly to the remote receiver. They proposed a health monitoring system for concrete structures using self-powered damage sensors, especially for detecting the cracks inside the concrete. A simple damaged beam example was used to illustrate how the harvester works and the performance of the sensor was analyzed theoretically and experimentally.

A cantilever-type energy harvester has been intensively studied, and a tapered cantilever has been found to be the optimum design by many researchers [7-10] because it ensures a large constant strain (and a large power output) in the piezoelectric film. Experimental verification is done for the tapered design by screen-printing a piezoelectric material (PZT-5H powder) onto 0.1 mm thick stainless steel [7]. Zheng et al. [11] conducted topology optimization of the piezoelectric energy harvester for the maximization of energy conversion efficiency. They found the optimal topology when a static force at the end of beam was applied. The adjoint method was used for sensitivity analysis and the material properties were parameterized using the SIMP method.

Typically, cantilever-type energy harvesters need to be tuned so that their resonant frequency is as close as possible to the excitation frequency in order to maximize power output. The performance of the harvesters may vary because the frequency of most ambient vibration has stochastic or random nature. This fact motivated the research for a new energy harvester with reliable energy harvesting performance regardless of the random nature of ambient vibration. The harvester considered in this paper is a cantilever-type which is excited at its free end by a rotary mechanical component. The tip excitation by contact with a rotating mechanical component can provide

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relatively larger deformations than base excitation, so one can expect larger strain rates and electrical energy output. Energy harvesters utilizing rotary motion are found in some recent studies. Khameneifar et al. [12] presented a piezoelectric bimorph cantilever with a tip mass mounted on a rotating hub. Hashimoto et al. [13] and Carroll [14] gave examples of a piezoelectric generator which utilized piezoelectricity to convert the relative motion between a rotor and a stator. Chen et al. [15] proposed a cantilever energy harvester of which the free tip is excited by a cam-shaft connected to a small wind turbine.

Most studies on the piezoelectric harvester for harvesting machine rotary energy used rectangular unimorph/bimorph cantilever-type harvesters, and no research has ever been done on the shape optimization of the piezoelectric harvester subject to free tip excitation based on the literature review performed. This study aims to improve piezoelectric energy conversion performance of a tip – excited cantilever harvester by the shape optimization technique. The averaged power output through a range of excitation frequencies is formulated as the objective function. The shape optimization results are compared with a rectangular cantilever and a tapered cantilever, and the prototypes are manufactured for performance evaluation.

This paper is organized as follows: The simulation model construction technique for a cantilever-type piezoelectric energy harvester is explained and verified in Section 2. The design optimization and the prototype evaluation are conducted in Section 3.

## 2. Simulation model construction and verifications

# 2.1 Cantilever-type piezoelectric energy harvester

The widely accepted piezoelectric energy harvester, a cantilever-type harvester, is briefly explained in this section. It utilizes the piezoelectric effect to generate electric potential in response to the applied mechanical strain. The governing equations of the piezoelectric effect are as follows:

$$S = s^{E}T + dE$$

$$D = dT + \varepsilon^{T}E$$
(1)

where S is the strain vector, T is the stress vector, E is the electric field vector, D is the electric flux density vector, s is the elastic compliance matrix, d is the material constant matrix, and  $\varepsilon$  is the permittivity.

A piezoelectric bimorph is a cantilever-type harvester composed of two piezoelectric layers. According to the polarization directions of the two piezoelectric sheets, the bimorph is categorized into serial connections and parallel connections as shown in Fig. 1.

Two piezoelectric sheets are attached on the neutral plane to have opposite polarization directions in serial connection while two sheets have the same polarization direction in paral-



Fig. 1. Polarization of bimorph.



Fig. 2. FE modeling of piezoelectric bimorph using ANSYS.

lel connection. When a serially-connected bimorph is deformed, the upper and lower surfaces generate opposing voltage so that the voltage difference is two times greater than the parallel connection. However, a parallel connection generates twice the current when compared to a single sheet. This is because the same phase currents are generated at each side of the closed circuit, and this property is beneficial for charging a storage unit (e.g., capacitor, rechargeable battery). The bimorph with a parallel connection is therefore chosen in this paper.

#### 2.2 Simulation model construction and verification

The finite element model for the bimorph energy harvester is constructed and verified in this section. For FE model construction, three different types of elements are used in ANSYS (see Fig. 2): SOLID5 for piezoelectric material, SOLID45 for the substrate, and CIRCU94 for external resistance where the power is measured. SOLID5 element contains four coupledfield degrees of freedom (DOFs) (three translational DOFs and one electrical potential DOF), and SOLID45 element for the substrate has three translation DOFs. Fig. 2 shows an example of FE modeling for a bimorph with parallel connection. A resistor is connected between the top electrode and the piezo-substrate interface (electrically grounded), and the top and bottom electrodes are additionally connected using a wire with a very small resistance value.

We verified our FE modeling technique by comparing with the work done by Erturk and Inman [16]. The FE model for the bimorph harvester is constructed using the model data in Table 1 and the external resistor is chosen as 1 k $\Omega$ . The voltage spectrum is compared in the excitation frequency range of  $30 \sim 70$  Hz. The validity of the FE model can be assured in Fig. 3 which shows an excellent agreement between the voltage spectrum by the FE analysis and the analytic solution available in Ref. [16].

Table 1. Geometric parameter and material properties for the bimorph cantilever [16].

| Material                |                                      | PZT-5A             | Substrate<br>(brass) |
|-------------------------|--------------------------------------|--------------------|----------------------|
|                         | Length<br>(mm)                       | 50.8               | 50.8                 |
| Geometric<br>parameters | Width<br>(mm)                        | 31.8               | 31.8                 |
|                         | Thickness<br>(mm)                    | 0.26(each)         | 0.14                 |
|                         | Tip mass<br>(kg)                     | 0.012              |                      |
| Material<br>properties  | Mass density<br>(kg/m <sup>3</sup> ) | 7800               | 9000                 |
|                         | Young's modulus<br>(GPa)             | 66                 | 105                  |
|                         | Strain coefficient<br>(pm/V)         | -190               | -                    |
|                         | Permittivity<br>(F/m)                | 1500ε <sub>0</sub> | -                    |



Fig. 3. FE model verification with the analytical solution.

#### 3. Design optimization of energy harvester

Based on the FE model in Section 2, the design optimization of the shapes of piezoelectric energy harvesters subject to tip excitation is studied in this section. The experimental verification for the optimized design will be performed using a gear rotating in a ball mouse. The length of the harvester is fixed at 19 mm due to limited space inside the mouse. The tip excitation speed is limited between 10 and 800 Hz and the tip displacement magnitude is set as 0.69 mm based on direct measurements. The maximum frequency (800 Hz) has been determined based on the approximated maximum mouse speed of 20 cm/s - at this speed the rotating speed of the mouse gear was observed to be 22~23 rpm and 36 teeth on the gear excites the free tip of the harvester at the frequency of about 800 Hz. Instead of metal, PET is chosen as the substrate so the wear of the mouse gear is minimized. The PET sheet is 2 mm longer than the piezoelectric sheets to provide a hitting space as shown in Fig. 4. PVDF is used for the piezoelectric

Table 2. Material properties for PVDF [17].

| Property                        | Units                      | Value     |  |
|---------------------------------|----------------------------|-----------|--|
| Strain coefficient $(d_{31})$   | 10 <sup>-12</sup> m/V      | 20~28     |  |
| Strain coefficient ( $d_{33}$ ) | 10 <sup>-12</sup> m/V      | -33~-30   |  |
| Coupling coefficient $(k_{3l})$ | CV/Nm                      | 0.11~0.12 |  |
| Coupling coefficient $(k_{33})$ | CV/Nm                      | 0.14~0.2  |  |
| Dielectric constant             | $\epsilon/\epsilon_0$      | 12~13     |  |
| Elastic constant                | $10^{10}$ N/m <sup>2</sup> | 0.2~0.4   |  |
| Tensile strength                | $10^{7}N/m^{2}$            | 4.5~5.5   |  |

Table 3. Material properties for substrate [18].

| Substrate | Young's modulus | Poisson's ratio | Density               |
|-----------|-----------------|-----------------|-----------------------|
| PET       | 1GPa            | 0.3             | 1.32kg/m <sup>3</sup> |



Fig. 4. Design variables in the initial model.

material for the conversion of large strain because it is less fragile and has 2.6 times larger tensile strength than PZT. PVDF sheets (17 mm long, 0.028 mm thick) are attached on both sides of the substrate to form a bimorph. Key material properties of PVDF and the substrate are summarized in Tables 2 and 3, respectively.

## 3.1 Optimization formulation

The output power of the piezoelectric energy harvester is chosen as the design objective, which is found by Eq. (2):

$$P = \frac{I_{R0}V_{R0}}{2} = \frac{I_{R0}^2 R0}{2} = \frac{V_{R0}^2}{2R0}$$
(2)

where  $I_{R0}$  is the output current of the piezoelectric energy harvester and  $V_{R0}$  is the voltage drop through the oscilloscope internal load ( $R0 = 1M\Omega$ ) as shown in Fig. 5.

The bimorph geometry in Fig. 4 is parameterized using a spline curve. The lateral sides of the plate are modeled with a spline curve with four equally spaced control points. Each design variable is defined as the distance between two facing control points between the two sides (DV1 to 4 in Fig. 4). The width of the initial design is 4 mm. The damping ratio is measured for the initial design by the impulse test [19] and the measured value (0.045) is assumed to be constant during the



Fig. 5. Piezoelectric energy harvester with internal resistance of oscilloscope.

# optimization.

The FE model is constructed as explained in Section 2: SOLID5 for piezoelectric material, SOLID45 for the substrate, and CIRCU94 for the internal resistance of the oscilloscope and wires. A total of 4116 elements are used for the bimorph model. At the clamped end, the 3 translational DOFs (degrees of freedom) are all fixed. The harmonic amplitude of the applied oscillation at the free tip was 0.69 mm.

The optimization problem is formulated as follows:

Find 
$$DV_j$$
 to  

$$\max \frac{1}{\omega^I} \int_0^{\omega^I} Pd\omega \approx \frac{1}{\omega^I} \sum_{i=1}^{N_f} \left( P^i \times \frac{\omega^I}{N_f} \right) = \frac{1}{N_f} \sum_{i=1}^{N_f} P^i$$
s.t.  $S = s^E T + dE$   
 $D = dT + \varepsilon^T E$  (3)  
 $\sigma \leq \sigma_{DTS}^0$   
 $3mm \leq DV_j \leq 6mm, \ j = 1,...,4$   
 $Area_{PVDF} \leq Area_{PVDF}^0$ 

where

| $N_f$                                     | : Number of bins for numerical integration in the       |
|---|---|
|   | Frequency domain  |
| $P^{i}$                                   | : Power of <i>i</i> th bin                              |
| $\sigma$                                  | : Maximum bending stress in PVDF                        |
| $\sigma^{\! ho}_{\scriptscriptstyle DTS}$ | : Tensile yield strength of PVDF (45MPa) [17]           |
| $DV_j$                                    | <i>j</i> th design variable                             |
| Area <sub>PVDF</sub>                      | : Area of PVDF ( $Area^{0}_{PVDF}$ is its upper limit). |
|   |   |

The objective function is the average power spectrum (*P*) in the frequency range of interest  $(0 \sim \omega^{I})$ , which can be calculated by numerical integration. In this design study,  $\omega^{I}$  is chosen to be 800 Hz and  $N_{f}$  is set as 80 (the sampling rate (or the bin size) is 10 Hz). The optimization is performed with or without the area constraint in Eq. (3) ( $Area_{PVDF} \leq Area^{0}_{PVDF}$ ), which corresponds to DESIGN 1 and DESIGN 2 in Section 3.2.

The optimization procedure is shown in Fig. 6. An FE model by ANSYS is used to calculate the performances required for optimization (e.g., the output power, the mechanical stress distribution). The optimization code is built using MATLAB which (i) reads the performance measures from the ANSYS output file and (ii) iteratively updates the ANSYS FE model based on the optimization algorithm. The SQP (Se-



Fig. 6. Optimization process.

quential Quadratic Programming) algorithm, a Newton's method applied to the solution of the KKT conditions, is chosen as the optimization algorithm. This method is widely used in design optimization research due to its superior rate of convergence [20]. The convergence criterion ( $\Delta f$ ) of the objective function is set to be 0.004 – the optimization procedure stops when the difference of the objective function values from two consecutive iterations is less than  $\Delta f$ .

#### 3.2 Optimization results

The optimization results are summarized in Table 4 in which the initial design (DESIGN 0) and two design cases (DESIGN 1 and 2) together with the well known tapered design (REF) are compared. Unlike the tapered design, the material is moved to the free tip for both of the optimized cases (DESIGN 1 and 2). DESIGN 1 has a narrower free tip than DESIGN 2 due to the area constraint. It is noted, against intuition, that DESIGN 2 does not converge to its allowed maximum area (all the design variables are 6 mm) even though there was no constraint on the area. This study verifies that the design with the wider free tip is the optimized shape for the harvester subject to tip excitation.

The output power spectra for each case are displayed in Fig. 7. The lower resonant frequencies in both DESIGN 1 and 2 are due to the increase in mass on the free tip. The power spectrum for each case is numerically integrated through the frequency domain of interest (10~800 Hz) to yield the average power in Table 4. According to Fig. 7 and Table 4, DESIGN 2



Fig. 7. Power spectra for DESIGN 0, DESIGN 1, DESIGN 2, and REF.

generates the largest power and REF generates the smallest. It is concluded that the design with a wider free tip improves the power output level, and the tapered harvester (REF) is not an optimal design for the tip excitation condition.

The optimization history on the objective function is shown in Fig. 8. 20 function calls are required through three iterations for DESIGN 1, while 30 function calls are required (five iterations) for DESIGN 2. It is observed that DESIGN 2 takes more iterations to find the solution in a larger feasible design space (because the area constraint is not considered) than DESIGN 1.



Table 5. Comparison of design and measured voltage waveform.

Fig. 8. Objective function converging histories.

## 3.3 Prototype fabrication and experimental evaluation

The piezoelectric energy harvester prototypes are fabricated for the results in Table 4 and installed in the test device (the ball mouse with a rotary gear) as shown in Fig. 9. The dynamic waveform of the voltage from the harvester is analyzed and the performance is measured as shown in Table 5 and Table 6. Table 5 compares the performance among the four prototypes in terms of the design shape and the voltage waveform, and the charged voltages and powers are listed in Table 6. The charging performance is measured using a  $10\mu$ F capacitor and the accumulated voltage is measured for one minute. We roll the mouse at a random velocity to excite each

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Table 4. Optimal shape: Max. average power.

| Design                                     | DESIGN 0 | DESIGN 1          | DESIGN 2          | REF              |
|--|----------|-------------------|-------------------|------------------|
| Average<br>charged<br>voltage <sup>*</sup> | 2.41V    | 2.82V<br>(+17.0%) | 3.59V<br>(+49.0%) | 2.48V<br>(+2.9%) |
| Average                                    | 0.483µW  | 0.663µW           | 1.073µW           | 0.513µW          |

 Table 6. Comparison of harvester performance.

\* Based on the charged voltage to 10µF capacitor in 1 minute. Averaged after 10 time measurements



Fig. 9. Mouse with piezoelectric energy harvesters.

harvester, and the measurements are done 10 times for each harvester.

Based on the accumulated voltages in the first row of Table 6, we can conclude that the optimized design (DESIGN 1 and 2) has better performance than DESIGN 0 or REF. The measured power values in the table are about three or four times smaller than the simulated results in Section 3.2 because the charging circuit is not designed to match the impedance with the simulation model, but the better performance of the wider tip design (DESIGN 1 and 2) still holds in this experimental verification. The accumulated electric energy  $(1/2 \times CV^2)$  is improved by 37% (39.8µJ) for DESIGN 1 and 122% (64.4µJ) for DESIGN 2 when compared to DESIGN 0 (29.0µJ).

# 4. Conclusion

In this research, the new design of piezoelectric energy harvester subject to tip excitation was proposed. The mechanical and electrical behaviors of piezoelectric materials were solved by coupled analysis using ANSYS, and the design optimization was performed for power maximization. Considering the randomness on the excitation frequency, the averaged power output through a range of excitation frequencies is formulated as the objective function. For the free tip excitation condition, this work found a new design for a cantilever-type harvester with wider free tip. The prototypes were fabricated and the performance of the proposed design was evaluated. In the proposed test device, the new optimal design showed about 37% improvement in terms of charged energy when compared to the rectangular shape using the same amount of material.

Even though the objective function is formulated considering the random speed of the gear rotation, a reliable and practical formulation needs to be considered where the statistical information on the rotation speed is included in the formulation. Design improvement on tip excitation devices to reduce friction and selection of materials with higher efficiency and durability remain as our future work for practical use of the proposed design.

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