Characterization of the ballistic limit curve for hypervelocity impact of sphere onto single plate†

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Abstract
A ballistic limit curve for hypervelocity impact of spherical aluminum projectiles on a single wall aluminum plate is examined using the smooth particle hydrodynamics (SPH) method. A brief description is provided about a new in-house code with an emphasis on the axi-symmetric coordinate scheme. The Benchmark Taylor bar impact test was first demonstrated as a validation of the code. Then a series of hypervelocity impact simulations was performed in axi-symmetric coordinate. The impact velocity ranged from 2 km/s to 10 km/s. The plate thickness varied from 2 mm to 8 mm. The ballistic limit results calculated are compared with predictions from empirical correlations, and the results are shown to fall within the envelope of the empirical correlation. A simple theory is developed to analyze the characteristics of ballistic limit curves. The theory provides a good insight on the hypervelocity plate impact event and is valuable for design concerns.

Keywords: Ballistic limit curve; SPH; Hypervelocity impact

1. Introduction
Understanding and controlling large plastic deformation problems such as hypervelocity impact/penetration (roughly \( V > 2 \) km/s) events are of great importance for protection of space vehicles and satellite system as well. This is because the external walls are exposed to impacts from meteoroid or space debris [1]. In these areas, experimental test has been conducted up to 7 km/s, and numerical simulations are performed for higher velocities. For protection, one uses dual-plate shields or multishock shields [2-4]. Details of the characteristics of debris clouds produced by hypervelocity impact of aluminum spheres with thin aluminum plates were described using flash radiographs [5].

Some studies were also conducted for a single plate impact case and several empirical equations for ballistic limit are available in the literature [6-8]. However, the equations are just fitting curves, hence the characteristics of hypervelocity plate impact is not well explained. In this paper we try to characterize the ballistic limit curves for hypervelocity impact of a sphere onto a single wall aluminum plate. This work is a combined numerical and analytical study. A series of simulations using the smooth particle hydrodynamics (SPH) code, which we recently developed, has been conducted. At the same time, by using a simple analytical model which we propose here, how significant parameters vary over a wide velocity range is provided.

The SPH formulation is stated in Section 2 with validation for the Taylor bar impact test. The hypervelocity plate impact simulation is discussed in Section 3. An analytical model is also included. The predicted ballistic limit curve is compared with the empirical equations and the characteristics are provided in detail.

2. Smooth particle hydrodynamics theory
2.1 Introduction
Lucy [9], Gingold and Monaghan [10] first introduced the smooth particle hydrodynamics (SPH) in space science areas. SPH is an interpolation method to calculate values and derivatives of continuous field properties by using discrete points which are located completely arbitrary. Because of the significant advantages over conventional grid based Lagrange scheme (no mesh distortion problem), SPH scheme was adopted and applied to the computations of large deformation problems [11, 12]. Johnson et al. [13] and Hayhurst et al. [14] implemented relatively simple algorithms for axi-symmetric geometry. These are much similar in that they consider each particle as torus ring geometry. The cylindrical SPH code we developed recently is much simple to migrate from a Carte-
sian coordinate version. A brief discussion of the mathematical description is provided below.

2.2 Mathematical formulation

The internal forces in solids with material strength are governed by the following conservation equations:

\[ \frac{Dp_i}{Dt} = \rho_0 \left( \frac{\partial u_i}{\partial r} + \frac{1}{r} \frac{\partial u_i}{\partial \theta} + \frac{u_i}{r} \right) \]

\[ \frac{Du_i^r}{Dt} = \frac{1}{\rho} \left( \frac{\partial \sigma_i^r}{\partial r} + \frac{\partial \sigma_i^\theta}{\partial \theta} + \frac{\sigma_i^r - \sigma_i^\theta}{r} \right) \]

\[ \frac{Du_i^\theta}{Dt} = \frac{1}{\rho} \left( \frac{\partial \sigma_i^\theta}{\partial r} + \frac{\partial \sigma_i^r}{\partial \theta} + \frac{\sigma_i^\theta - \sigma_i^r}{r} \right) \]

in cylindrical coordinates,

\[ \frac{DP}{Dt} = \rho \left( \frac{\partial u_i}{\partial x} + \frac{\partial u_j}{\partial y} + \frac{u_i}{r} \right) \]

\[ \frac{Du_i}{Dt} = \frac{1}{\rho} \left( \frac{\partial \sigma_i^r}{\partial x} + \frac{\partial \sigma_i^\theta}{\partial y} + \frac{\sigma_i^r - \sigma_i^\theta}{r} \right) \]

\[ \frac{Du_j}{Dt} = \frac{1}{\rho} \left( \frac{\partial \sigma_j^r}{\partial x} + \frac{\partial \sigma_j^\theta}{\partial y} + \frac{\sigma_j^r - \sigma_j^\theta}{r} \right) \]

where \( \sigma \) is the stress tensor and \( u \) is the velocity. \( x, y \) are the Cartesian coordinates and \( r, \theta \) are the cylindrical coordinates in radial- and axial direction. Note that two sets of equations are much similar except some extra terms added in the cylindrical coordinates, such as a hoop stress in the radial acceleration. These equations are to be switched depending on the geometry chosen. A commonly used expression, in the Cartesian coordinates, which is converted to a particle approximation via a kernel function, \( W \), is:

\[ \frac{Dh_i}{Dt} = \sum_{j=1}^{N} m_i \left[ \left( u_i^r - u_j^r \right) \frac{\partial W}{\partial r} + \left( u_i^\theta - u_j^\theta \right) \frac{\partial W}{\partial \theta} \right] \]

\[ \frac{Du_i^r}{Dt} = \sum_{j=1}^{N} \left[ \frac{\sigma_i^{rr} - \sigma_j^{rr}}{\rho_i} \frac{\partial W}{\partial r} + \frac{\sigma_i^{r\theta} - \sigma_j^{r\theta}}{\rho_i} \frac{\partial W}{\partial \theta} \right] \]

\[ \frac{Du_i^\theta}{Dt} = \sum_{j=1}^{N} \left[ \frac{\sigma_j^{r\theta} - \sigma_j^{r\theta}}{\rho_i} \frac{\partial W}{\partial r} + \frac{\sigma_j^{\theta\theta} - \sigma_j^{\theta\theta}}{\rho_i} \frac{\partial W}{\partial \theta} \right] \]

where \( N \) is the number of pairs which are identified to exist within the smoothing length (influence zone), \( h \). Here \( i \) and \( j \) represent the center particle and the neighbouring particles. The smoothing function value is obtained by the summation of the neighbouring particles where the influence is varying with the distance. The most widely used cubic B-spline smoothing function is expressed as

\[
W(R,h) = c_d \begin{cases} 
\frac{3}{2}h^3 - R^2 + \frac{1}{2}h^3 & 0 \leq R < 1 \\
\frac{1}{6}(2-R)^3 & 1 \leq R < 2 \\
0 & 2 \leq R 
\end{cases}
\]
In cylindrical coordinates:

\[ f(x) = \int f(x) W(x-x',h) dx \]

\[ = \int f(x) W(x-x',h) r dr dz \]

\[ \approx \sum_{j=1}^{N} m_j f(x_j) W(x-x_j,h) \Delta V_j \]

\[ = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W(x-x_j,h) \]

\[ \approx (2\pi r_j)(\Delta z)(\Delta y). \]

As demonstrated in Eq. (6), in Cartesian coordinates the unit of \( W \) should be the inverse of two-dimensional volume and Eq. (4) satisfies this requirement. In the cylindrical SPH, however, if we use the same coefficient in smoothing function mismatch occurs in this approximation since \( dV/m \) is in fact a three-dimensional volume (mass). A remedy can be included directly into the calculation of the smoothing function as below:

\[ W_{axisymmetric} = \frac{W_{0}}{(2\pi r_t)^2} \]

Now the unit consistency is recovered, and at the same time it is guaranteed that the neighbouring particle (j) at large radial location has bigger volume (mass) by the factor of \( r_j/r_t \). The same is true to the particle approximation for the spatial derivative of the function values. In conclusion, this method allows us an easy incorporation of the cylindrical SPH scheme since it is not necessary to change any other equations used in the Cartesian SPH formulation.

The validation of the current SPH code has been conducted for the Taylor bar impact test. Both three-dimensional and axisymmetric simulations are performed and results are compared with experimental data. The discussion is included in the Appendix.

3. Hypervelocity impact simulations

3.1 Summary of empirical models

There are some empirical models for ballistic limit curves [15]. For a characterization purpose, at the same time for convience, a short description is described below. The model provides the threadhold limit (ballistic limit) for perforation of thin metal plates. That is, they are used to determine either the plate thickness to prevent a penetration or the critical sphere diameter.

Fish-Summers model

This is an empirical model proposed by Fish and Summers with velocities from 0.5 to 8.5 km/s. It is a simple form and material properties are inclusive in a constant \( K_i \).

\[ t = K_i m^{0.555} V^{-0.875} \rho^{1.16} \]  

(8)

where

\( t = \) Plate thickness (cm),
\( K_i = \) A constant for plate (0.57 for aluminum alloys),
\( m = \) Sphere mass (gm),
\( \rho = \) Sphere density (gm/cm^3),
\( V = \) Impact velocity (km/s).

Schmidt-Holsapple model

This is another equation developed by Schmidt and Holsapple using test data with velocities from 4.0 to 8.5 km/s. The sphere materials are tungsten, carbide, lead, copper, stainless steel, magnesium and aluminum and plate materials are stainless steel and aluminum.

\[ d = 2.06 \left( \frac{\rho_p}{\rho_t} \right)^{0.159} \left( \frac{2.68 F_{tu n}}{\rho_p V_n} \right)^{0.236} \]  

(9)

where

\( d = \) Sphere diameter (in),
\( t = \) Plate thickness (in),
\( \rho_p = \) Sphere projectile density (lb/in^3),
\( \rho_t = \) Target plate density (lb/in^3),
\( F_{tu n} = \) Ultimate tensile strength for plate (lb/in^2),
\( V_n = \) Impact velocity (normal component) (ft/s).

Rockwell model

This following equation is an empirical equation obtained using test data with velocities up to 8.0 km/s. Both sphere and plate are aluminum.

\[ t = (1.8 \times 1.38) d^{1.1} (BH) 0.25 \left( \frac{\rho_p}{\rho_t} \right) 0.167 (V)^{2.3} \]  

(10)

where

\( d = \) Sphere diameter (cm),
\( t = \) Plate thickness (cm),
\( \rho_p = \) Sphere projectile density (gm/cm^3),
\( \rho_t = \) Target plate density (gm/cm^3),
\( BH = \) Brinnell hardness for plate,
\( V = \) Impact velocity (km/s).

Modified JSC model

This is the most recent equation.

\[ t = (1.8 \times 5.24) d^{1.1} (BH) 0.25 \left( \frac{\rho_p}{\rho_t} \right) 0.5 \left( \frac{V}{c} \right)^{2/3} \]  

(11)

where

\( t = \) Plate thickness (cm),
\( c = \) Speed of sound (km/s).
3.2 Hypervelocity impact simulation results

3.2.1 Modeling

A series of SPH simulations has been conducted for impact of an aluminum sphere on a single wall aluminum plate at normal incidence. All was performed in two-dimensional axisymmetric domains. The impact velocity was parametrically varied in a series of numerical simulations from 2 km/s to 10 km/s. The plate thickness considered here ranged from 2 to 8 mm. The critical sphere diameter are found to be between 0.5 mm and 1.5 mm. We tried to maintain a consistent mesh resolution system in all cases. For the plate, particle size varies from 0.133 mm to 0.125 mm such that the plate is packed with 15 or 16 particles across the thickness. The radial extent of a plate is 12.5 mm which is approximately 10 times of the largest sphere radius. The size ratio of sphere particle to plate particle is less than one, roughly 0.7–0.8, but is attempted to be of comparable sizes.

The constitutive response for 6061 T6 Aluminum was represented by the Johnson-Cook model [16],

\[
\sigma = \left[ 150 + 113(e^*)^{0.2} \right] \left[ 1 + 0.002 \ln \left( \frac{e^*}{0.1} \right) \right] \left(1 - T^\Gamma \right) \quad \text{(MPa)}
\]

where \(\sigma\) is the von Mises effective flow stress (MPa), \(e^*\) the equivalent plastic strain, \(e^*\) the dimensionless plastic strain rate, and \(T^\Gamma\) the homologous temperature. The values used for the equation of state are also listed in Table 1, where \(c\) is the speed of sound, and \(G\) the shear modulus.

3.2.2 Phases of penetration

Simulated impact processes for impact of a sphere onto a thin aluminum plate at the impact velocity of 6 km/s are shown in Fig. 2. This is the no-penetration case. Much of the simulation seems to be reasonable, showing plastic deformation and bulging at the rear surface. The penetration velocities predicted at the particles located at front (point 1) and rear (point 2) surfaces, along the centerline, are shown in Fig. 3 for \(V = 6\) km/s. At point 1, there is an initial transient (shock) phase, indicating an abrupt deceleration, while the particle at point 2 experiences no deceleration. It is followed by a moderate deceleration phase. At point 2 a short steady penetration phase is predicted between them. This steady state penetration phase in which penetration velocity is approximately constant with time becomes more dominant if the impacting body is long (a Taylor bar that is \(L/D > 5\)). Here, \(L\) is length and \(D\) diameter [17].

The quick deceleration at rear surface of a sphere which is caused by stress waves is finished less than \(t = 0.1\) us after impact. A first order estimation can be made here. Plastic waves generated at front surface start to propagate to rear surface. If the wave speed is approximately 5.5 km/s, then the wave reaches the rear surface (0.64 mm diameter) within 0.11 \(\mu\)s.

\[
\text{time} = \frac{D}{c} = \frac{0.00064(m)}{5,500(m/s)} = 0.116\mu\text{s}
\]

Table 1. Material properties for the aluminum.

<table>
<thead>
<tr>
<th>(\rho) (kg/m(^3))</th>
<th>(c) (m/s)</th>
<th>(S)</th>
<th>(\Gamma)</th>
<th>(G) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2750</td>
<td>5350</td>
<td>1.339</td>
<td>2.0</td>
<td>25</td>
</tr>
</tbody>
</table>

- Linear shock velocity: \(u_s = c + S u_p\)
- \(\Gamma\): Gruneisen constant, \(\frac{\rho}{c^2}\)

\(d\) = Sphere diameter (cm), \(\rho_p\) = Sphere projectile density (gm/cm\(^3\)), \(\rho_t\) = Target plate density (gm/cm\(^3\)), \(BH\) = Brinnell hardness for plate, \(E\) = Young’s modulus (GPa), \(V_n\) = Impact velocity (normal component) (km/s), \(c\) = Speed of sound for plate (\(\sqrt{E/\rho_t}\)) (km/s).

**Fig. 2.** Simulated impact of a sphere \((d = 0.5\) mm\) on AL 6061-T6 plate for 6 km/s, \(t = 0.0, 0.1, 0.5, 1.0\) \(\mu\)s after impact.

**Fig. 3.** Deceleration of a sphere after plate impact. Point 1 is the particle at the center of front surface and point 2 is the particle at the center of back surface.
3.2.3 Critical diameter versus impact velocity

In this section, the dependence of critical sphere diameter on impact velocity is examined for 2 mm aluminum plate. The impact velocity varies from 2 km/s to 10 km/s. Even in order to predict one critical diameter for each impact velocity, several runs are carried out by changing the sphere diameter with fine resolution of ± 0.005 mm. It can be expected that as the impact velocity increases, the critical diameter becomes smaller.

Fig. 4 shows the minimum sphere diameter predictions as a function of impact velocity. Also shown are predictions from the empirical models. Since we are interested in the determination of the ballistic limit, spall failure criteria are not specified in the simulations. The empirical models also provide the ballistic thresholds. The parameters used in the models are the ultimate tensile limit \( F_{tu} = 250 \text{ MPa} \) (Schmidt-Holsapple Model) and Brinell hardness for plate BH = 95 (modified MSC model). The current SPH predictions are fairly well within the envelope given the empirical results, and seem to agree most closely with the JSC model. The simulation results follow the empirical curves for a wide range of velocity.

In order to have some insights on the parameter dependency in the ballistic limit curve, an analysis which is based on the energy conservation argument is provided below [15]. That is, as shown in Fig. 5, the total energy of impacting body is equal to the work used for perforation of a plate of thickness \( t \),

\[
\frac{1}{2} m V^2 = Y * (\text{Perforated volume}) = Y * (Ad)
\]  

(14)

where \( Y \) is the yield strength, \( A \) the area of perforation hole, and \( m \) mass of impacting sphere which is proportional to \( d^3 \). For fixed plate thickness, the work required to perforation is assumed to be constant. This seems to be reasonable as a first order estimation.

\[
d^3V^2 = C_1^3
\]  

(15)

Then this can be rewritten as,

\[
d \propto \left( \frac{1}{V} \right)^{2/3}
\]

(16)

Now we examined the equations provided from the empirical models in some details. Table 2 shows comparison of the dependence of the minimum diameter on impact velocity while plate thickness is fixed. The exponent values \( n \) are slightly different between models and the value 0.67 in Eq. (16) is within bounds. The Fish-Summers model shows the worst dependency, as indicated in Fig. 4.

3.2.4 Critical diameter versus plate thickness

In this section, the variation of minimum sphere diameter is analyzed as a function of plate thickness while the impact velocity is fixed at 6 km/s. The plate thickness varies from 2 mm to 8 mm. Even one critical diameter for each impact velocity can be obtained with several runs by changing sphere diameter. Again the diameter resolution is ± 0.005 mm. Fig. 6 shows the minimum sphere diameter predicted by SPH simulations. The critical diameter increases with increasing plate thickness, indicating a linear correlation. The slope is found to be 0.356.

An explanation by using the previous energy conservation argument is provided below. We go back to Eq. (14). In this case \( V \) is fixed, but the plate thickness is not. Since \( A \) (perforated hole area) is proportional to \( d^2 \), and \( m \propto d^3 \), Eq. (14) can be rewritten as

\[
d = C_2 t \Rightarrow d \propto t
\]

(17)

which indicates a linear correlation. We examined the empirical...
4. Conclusion

A ballistic limit curve (threshold penetration to given plate thickness) for hypervelocity impact of an aluminum sphere onto a single wall aluminum plate is investigated using the in-house code of smooth particle hydrodynamics (SPH) scheme. The numerical formulations are first described with an emphasis on the unique migration technique from a Cartesian coordinate into axisymmetric coordinate. Benchmark calculations for Taylor bar impact problem are demonstrated using both three-dimensional and axisymmetric geometries, and an excellent agreement with previous experimental data are obtained. Then a series of hypervelocity plate impact simulations was performed in axisymmetric coordinate. The impact velocity ranged from 2 km/s to 10 km/s and the plate thickness from 2 mm to 8 mm. The ballistic limit results calculated are compared with predictions from empirical models which are available in the literatures, and they are found to fall within the envelope of the empirical correlations. In order to analyze the characteristics of ballistic limit curves, we proposed a simple theory based on the energy argument. The theory provides a good insight on the hypervelocity thin plate impacts. It explains the relationship between the sphere critical diameter with plate thickness at given impact velocity and that between the sphere critical diameter with impact velocity at given plate thickness as well.

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References


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